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ABSTRACT

Presented are 13 articles which contain abstracts and analyses of research reports related to mathematics education. The first article considers six working papers, on a variety of topics, from E. G. Begle and the Stanford Mathematics Education Study Group. Of the remaining reports, two deal with problem solving, five with some aspect of learning style or theory, and one each with influences on pupil performance teacher effectiveness, a strategy for teaching indefinite integration, Piagetian tasks as predictors of achievement and teaching style. Research related to mathematics education which was reported in RIE and CIJE between July and September 1979 is listed. (MK)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Winter 1980

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SIX WORKING PAPERS FROM E. G. BEGLE AND THE STANFORD MATHEMATICS EDUCATION STUDY GROUP:

Begle, E. G. LEARNING CORRELATES FOR MEXICAN-AMERICAN STUDENTS. SMESG Working Paper No. 12, November 1972. ERIC: ED 142 406.

Begle, E. G. MATHEMATICS READING ABILITY. SMESG Working Paper No. 1, May 1973. ERIC: ED 142 415.

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Begle, E. G.; Perl, T.; Pence, B.; and Ng, K. THE EFFECTS OF VARYING THE NUMBER OF PRACTICE PROBLEMS, NUMBER OF EXAMPLES, AND LOCATION OF PRACTICE PROBLEMS IN ELEMENTARY SCHOOL GEOMETRY. SMESG Working Paper No. 16, April 1976. ERIC: ED 142 408.

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Abstract and comments prepared for I.M.E. by SUZANNE K. DAMARIN, The Ohio State University.

1. Purpose

In his work with SMSG, NLSMA, and other projects, E. G. Begle studied many variables and their potential relationships to mathematics learning. His recently published book (Begle, 1979) outlines his thoughts on areas in which fruitful research might be conducted and on questions which should be answered. The papers abstracted here represent efforts by Begle and his collaborators to examine some of these questions. The studies are diverse in their purposes, but to some extent they all grow out of the work of SMSG.

Three of the studies (Begle et al., 1975; Begle et al., 1976; Pence and Begle, 1974) concern the effects of number and position of examples and problems within instructional materials. Experimental psychologists have studied number and position of examples as independent variables in

abstract concept learning tasks. The three studies abstracted here were designed to relate these laboratory studies to classroom learning.

A study concerning the relationship of mathematics reading ability to mathematics aptitudes and achievements (Begle, 1973) used a Cloze test which was developed by SMSG and has been used in several studies. This investigation of reading ability at the fifth-grade level was intended to complement and extend an earlier similar study at the eighth-grade level.

Another survey study (Begle, 1972) was designed to examine the performance of Mexican-American students on selected SMSG measures and to compare this performance with that of largely Anglo-American populations.

A study concerning student achievement in the area of fractions (Ginther et al., 1976) was designed to provide information relevant to the debate over whether current teaching of fractions is adequate, and if not, whether the teaching of fractions should be postponed until secondary school.

2. Design, Procedures, Findings, and Interpretations of Studies

A. Three studies on effects of number and placement of examples and problems on student learning

These studies (Begle et al., 1975; Begle et al., 1976; Pence and Begle, 1974) all concerned the effects on learning of the number of worked examples and practice problems within programmed instructional units on mathematical topics. The studies were conducted at grades 5 and 6 (geometry and polygons), grade 7 (probability), and high school (negative bases). In each study, a battery of pretests was given prior to treatment and programmed units, differing only in the number or placement of examples and/or practice problems, were administered to randomly assigned students. Achievement data were collected by posttest or by tests internal to the programmed units. For each study, descriptive statistics and regression coefficients were computed. In the studies of probability and negative bases, analyses of covariance were also performed.

The basic finding in the two experiments involving instruction of fifth and sixth graders on geometry (Begle et al., 1976) was that neither the number and placement of practice problems nor the number of worked examples had a significant effect on student achievement.

The experiment on teaching probability to seventh graders (Pence and Begle, 1974) indicated that there might be some difference in learning attributable to the number of practice problems provided. This experiment used a posttest consisting of a relevant concept scale, an irrelevant concept scale, and a computation scale. The ANCOVA indicated that the presence of many practice problems contributed to success on the relevant concept scale (p less than .06) and that scores on the irrelevant concept scale were improved both by increased practice (p less than .11) and increased number of examples (p less than .09). When the population was divided into low and high groups on the basis of the Necessary Arithmetic Operations pretest, it was found that many practice problems produced higher scores on the relevant concepts scale among the low group (p less than .03).

In the third study of the effects of varying the number of practice problems (Begle et al., 1975), 421 high school students were given programmed instruction on negative bases. The students were classified into three groups on the basis of the number of mathematics courses completed and given a battery of pretests on numeration, arithmetic reasoning, bases, and problems. They were then assigned to four-day treatments in either the many practice problems or few practice problems group. Posttests included a word association test and a negative bases achievement test consisting of three scales: computation, comprehension, and transfer. Data analysis revealed that mean posttest scores increased with the number of mathematics courses completed and that the many problems treatment was more effective than the few problems treatment. For the group with the least mathematics, there were significant differences between treatments on all three achievement scales. For the group with the most mathematics, having many examples appeared to affect achievement significantly only on the comprehension scale.

B. Mathematics reading ability

Early in the Fall, all fifth graders in the 15 schools of one district were given a pretest battery consisting of a rote computation test of division, four simple geometric questions, a selection of non-routine problems, and an arithmetic reasoning test (Begle, 1973). A week later, the students were given a Cloze test on reading mathematics. Near the middle of the school year, computation (division) and problem solving posttests were administered. Correlations between all pairs of variables were computed. Multiple regression showed that the ability to read mathematical prose contributed substantially to problem solving and less so to computation posttest scores. However, the best predictors of mid-year scores on problem solving and computation were the fall scores on the respective comparable tests. The investigator observes that, although a correlation between Cloze test score and mathematical ability scores have been established, no cause-and-effect relationships are demonstrated.

C. Learning correlates for Mexican-American students

In this follow-up to an earlier study in which two versions of a programmed unit on primes and factorization of whole numbers were given to 100 primarily Anglo students, the same treatments were given to 35 Mexican-American seventh graders (Begle, 1972). Four pretest measures (Five dots, Necessary Arithmetic Operations, Division, and Factors and Primes) and one posttest measure were given in each study. The Mexican-Americans in the follow-up study scored slightly lower on each measure than the original population, but the mean scores were well above the chance level. Although most correlations among the measures were above .40 for the original population, none of the correlations between pairs of the five measures was significantly different from zero for the Mexican-American population. In particular, none of the pretest measures predicted success on the posttest for the follow-up population. These results demonstrate again the danger in generalizing to other cultures from studies of the ways Anglo-American children learn.

D. A survey of student achievement with fractions

Each of the 78 fraction items used by NLSMA in 1976 was classified as computation, comprehension, or application (Ginther et al., 1976). These items were distributed among 13 test forms of less than 10 items

apiece. Each test was administered to a population (average $N=193$) of average ability eighth graders. The percentage of students answering each item correctly was tabulated across all populations given the item, and these percentages are reported for selected items. The items on which data are reported were chosen as representatives of the more difficult and less difficult items in each of several categories.

The results showed that between one-half and two-thirds of the population performed computations involving $+$, $-$, and $\times$ correctly, but only 35 to 40 percent divided correctly. On comprehension items, 40 percent were able to use the missing factor definition of fraction, and 50 to 60 percent the number line. The percentage correctly interpreting shaded figures varied with the complexity of the figure. Performance on application problems was similar, with correct responses given by one-third to two-thirds of the population.

The authors conclude that even in the best circumstances (i.e., suburban schools serving average and above average students), the schools are not adequately teaching children to use fractions; they recommend that the study of fractions be postponed until secondary school.

Abstractor's Comments

1. These papers provide a valuable extension of earlier work with which Begle was associated. In comparison with the work of SMSG and NLSMA, the studies are generally small in scope; but this is part of their value, for these studies address particular issues which emerged as unanswered questions in the earlier work. Most of the pre- and post-test measures are drawn from earlier studies, and can therefore generally be interpreted with confidence.
2. Unlike journal articles, there are no generally accepted standards against which working papers should be evaluated or criticized. The name working papers implies that in some sense they are incomplete or inconclusive. Nonetheless, the availability of such papers can be a boom to the profession if they make available useful ideas and data. These six papers definitely serve this function. However, they would

have served it better if the numerous statistical tables (reproduced computer output) had been indexed and accompanied by a modest amount of interpretive information (e.g., translations of missing word labels, identification of base (test or subscale?) on which biserial r 's were computed, etc.). Had these authors indexed their tables they would surely have found and corrected the occasional instances of missing test data.

3. A more serious problem than missing data is present in the study of examples in probability instruction (Pence and Begle, 1974). The posttest scale testing irrelevant concepts is discussed as a dependent variable without mentioning that its reliability was only .019. The question of how the mixture of irrelevant concept items among relevant concept and computation items affects performance on the other items is an interesting one.

4. In his recent book, Begle (1979) has surveyed massive amounts of research and distilled from it a set of critical variables for future research in mathematics education. It would probably be a worthwhile endeavor for someone to perform a similar survey of the work that Begle has been involved with in order to generate or evaluate some hypotheses in relation to these critical variables. Taken together, the three papers on number of examples or problems, for example, suggest that the value of examples beyond the first two may be related to a type of mathematical sophistication which is measured partially by number of courses taken and partially by success in mathematical work. For another type of example, these six papers include one on Mexican-American students and another (Begle et al., 1976) in which a footnote refers to performance of Mexican-Americans on a different test. If all the references and footnotes relating to Mexican-Americans were culled from Begle's work and examined together, we might have a basis for beginning a sound investigation of some cultural differences in the learning of mathematics.

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Callahan, Hubert and Bell, Frederick. THE EFFECT OF SPECIALLY CONSTRUCTED ADVANCE ORGANIZERS AND POST-ORGANIZERS ON MATHEMATICS LEARNING. International Journal of Mathematical Education in Science and Technology 9: 433-450; 1978.

Abstract and comments prepared for I.M.E. by JOHN W. GREGORY, University of Florida.

1. Purpose

The purpose of the investigation was to determine if the use of a written advance organizer or post-organizer produced significant effects on immediate and delayed mathematics learning.

2. Rationale

Based upon the varying results of and problems associated with previous mathematics educational research concerning the use of advance and post-organizers, the study sought in particular to define clearly and conduct cleanly an investigation of this type. In particular, the study sought to utilize free-recall data in an effort to reduce the effect of pre- and post-measures functioning as advance or post-organizers.

Construction of instruments and materials as well as theoretical foundation for the investigation was based upon the work of Ausubel, which has found that expository instruction could be enhanced through the presentation of an advance organizer; that is, an oral or written statement presented prior to the learning of materials at a higher level of abstraction, generality, and inclusiveness.

3. Research Design and Procedures

The sample consisted of 57 male and three female undergraduate engineering students enrolled in a required statistics course. The subjects were randomly assigned to one of four treatment conditions created by a 2x2 factorial design of advance organizer (treatment and control) crossed with post-organizer (treatment and control).

Both treatment variables were written passages of approximately 300 words, read by the subjects immediately prior to or immediately following a 1½ hour lecture involving the central limit theorem. The

advance organizer passage (AO) contained references, made by common name, to 20 prerequisite concepts and principles (cognition labels). The post-organizer (PO) was a restatement of AO in language of a summarizing nature. The control for both pre- and post-lecture organizers consisted of a historical passage (HP) of approximately 300 words, not related to the learning content of central limit theorem.

Five measures of mathematical knowledge were obtained. The SAT mathematics score (SAT-M) served as a quantitative regressor. A 16-item objective test (LT) measured immediate learning of content presented in the lecture. Measures of "content recognition" required students to write a two-page passage in which each of the 20 cognitive labels (given to students in a random listing) referred to above was to be used at least once. This "free-recall" measure was obtained two days prior to the pre-lecture treatment (FR1), immediately following the post-lecture treatment (FR2), and two weeks following the lecture presentation (FR3).

Three mathematicians independently rated each of the 180 FR passages for evidence of correct recall of the 12 points presented during the lecture. FR's were rated from 1 to 13, with 1 indicating no reference to the lecture content up to all 12 points of the lecture represented (score of 13). The investigators posited that the cognitive labels about which the students were to write would prompt factual recall of the lecture content. The sum of the three ratings for each passage constituted a score for that FR.

The experimental sequence consisted of FR1 (Day 1); AO or HP treatment, lecture, PO or HP treatment, FR2, LT (all on Day 3); and FR3 (Day 17).

Data were submitted to linear regression analyses to test null hypotheses involving full and reduced linear models showing each of the effects (main and interactive). Non-parametric analyses were employed for assessing main and interactive effects where a linear model was not found significantly ($p < .05$) to describe the data.

4. Findings

In only one case (LT) did the general linear model provide a significant fit for the data. For this objective learning test measure, only SAT-M was found to have a significant effect. The entire seven-part linear model led to explaining 23 percent of the LT variability.

All scores for the FR1 measure were 1, indicating no evidence of any of the 12 points central to the lecture content. The investigators report this as evidence that no subject had previous knowledge of the topic of the study.

Scores by individual subject and each cell median for both FR2 and FR3 are reported. Scores ranged from 3 to 18 with overall mode and median for each of FR2 and FR3 equal to 3. This indicates that most subjects included no reference to any of the 12 major points of the lecture in writing their cued response passages. Cell medians equalled 3 except for the AO x PO and HP x PO cells, in which cases the medians both equalled 5. A score of 5 would mean that two of the judges each thought that one point of the lecture was present and the third judge rated the passage as having no reference to the lecture content.

Mann-Whitney large sample U procedures led to finding no significant differences in FR2 distributions between AO and HP (the two pre-treatment levels). Significant differences between post treatment levels of PO and HP for FR2 were found. No significant differences were found to exist within either treatments for FR3 performance rating sums.

No significant Spearman rank-order correlations were found between LT and either FR2 or FR3. Strong positive correlations were found to exist between FR2 and FR3 for each factor level of each treatment.

Interactive effects were not tested due to the finding of non-homogeneity of variance and non-normality of distribution of data, conditions needed for computing the linear model coefficients.

5. Interpretations

Within the constraints of the specific organizers and measuring instruments employed in the study, the investigators made the following conclusions:

- a. An advance organizer has no effect on mathematics learning of content presented in a college-level expository environment.
- b. A written post-organizer has a significant effect on immediate free-recall of lecture content, but not on immediate learning as measured by an objective test or on long-term free-recall.

The investigators interpret further that:

- a. The post-organizer offered an opportunity for rehearsal of the lecture content which aided short-term memory.
- b. Since the LT was organized externally to the learner using syntax chosen by the investigators, this instrument might itself have functioned as a post-organizer.
- c. The data support the conjecture that the cues used in the free-recall instrument elicit responses which are tempered by a greater degree of cognitive mediation than do externally organized objective questions such as were used on an objective test of recognition.
- d. It appears that new mathematical material which is presented in familiar terms during a lecture is itself performing some type of built-in organizational function in a cognitive structure, as Ausubel has predicted.

Abstractor's Comments

The investigators should be commended for their efforts to extend the research knowledge concerning learning effects due to organizers. In particular, the uniqueness of the study in relation to using free-response measures provides a model for future studies.

The procedures, findings and conclusions do lead to what this analyst intends to be constructive questions. These questions include:

- 1. On the basis of such low performance might on FR2 and FR3, could it be that revising the list of cognitive labels might lead to better key ideas which would prompt more factual recall of the lecture material? Similarly, might analysis of the FR passages for correct utilization and content relative to the cognitive

labels lead to some explanation of the low number of references made to the lecture content?

It could be that the uniqueness of the free-recall instrument led to a limiting effect on student performance on FR2 and FR3. The instructions to the students were to write a two-page passage in which each of the 20 cognitive labels was to be included. Literal interpretation of the instructions, combined with the relatively large number of labels to be included, could explain why the majority of the students made no reference to the lecture content on either FR measure. For those students who received either AO or PO treatments (or both), might they have modeled these passages in constructing their own since the same 20 cognitive labels were used? For all subjects, having performed on FR1 prior to hearing the lecture might have established a model for responding to the 20 cognitive labels which would necessarily be a model void of any reference to the lecture content.

As a suggestion, a smaller number of cognitive labels, revision of instructions for completing FR passages to include some student reflection upon the lecture content, and elimination of the pretest (since Ss were randomly assigned, preconditions were controlled) might lead to higher scores on post-lecture free-recall measures.

2. Although normality and homogeneity of variance assumptions would have been violated, might not a more robust statistic have been employed in order to ascertain main and interactive effects? The two-way ANOVA, for example, has been found to be robust enough to be minimally affected by such violations.
3. Might there have been a "cognitive overload" created by having students read a 300-word passage, listen to a one-and-one-half hour lecture, read another 300-word passage, write a two-page passage, and take an objective test, all in one sitting? Although the report does not include information regarding the performance of the students on the objective test (LT), would fatigue enter into the performance level of the students at this stage of the experiment?

In order to reduce the intensity of requirements placed upon students, future studies might consider reducing the number of major points in a lecture from 12 to 3 or 4. Also to be considered would be allowing students some time between the various parts of the study. In typical college learning situations, students are not tested immediately following a lecture. Since students had no opportunity to ask questions or review notes, the quality of the lecture (in terms of clarity and strategy of presentation) is a significant variable which can be both a basis for investigation as well as a source of variance in student performance.

4. The questions raised in Items 1 and 3 above reflect concern for the investigation's deviation from the "usual" in a college statistics course. Taking tests immediately following a lecture, no student-teacher or student-student interactions during the lecture, performing on unique instruments and possibly having a guest lecturer (the report does not indicate who presented the lecture) all deviate from "normal routine."

This concern does not refer to possible Hawthorne effects. The concern is, instead, for the possible need on the part of students to alter both their levels of expectation and their learning styles in order to accommodate the lecture and testing procedures. Future investigations might consider student performance over a series of lectures which are preceded and/or followed by organizers, as opposed to expecting significant effects after only one such lecture.

Claxton, G. and Acres, F. DOES HOW YOU LEARN MATH INFLUENCE HOW YOU FORM CONCEPTS. British Journal of Educational Psychology 48: 79-83; February 1978.

Abstract and comments prepared for I.M.E. by ROLAND F. GRAY,
University of British Columbia, Vancouver.

1. Purpose

By inference from the introductory statement, the purpose of this study was to examine the effects of two methods of teaching elementary school mathematics ... one a discovery method (Dienes), the other expository (Fletcher) ... on the development of concept formation skills.

Five hypotheses were tested. They were:

- a) Children taught by the Dienes' method will perform better on concept formation tasks.
- b) The differential favoring children learning by the Dienes' method will increase with time.
- c) The differential between Dienes and Fletcher children (sic) will be greater for real life than for abstract stimuli.
- d) Girls will perform better than boys.
- e) There will be a positive correlation between intelligence and performance on the tasks.

2. Rationale

Dienes is quoted as asserting that his "dynamic search activity" method of teaching mathematics aims at development of skills for formation and manipulation of concepts, particularly abstract concepts. Other authorities are cited who have stressed the difficulties inherent in evaluating discovery learning and who have pointed to the need for studying effects over long periods of time (Bruner, Whitrock, and Shulman and Keislar). Consequently, this study was designed to study the effectiveness of discovery-type learning over three- and six-year periods.

3. Research Design and Procedures

a. Sample

Two equivalent groups of children were selected from second-year classes and fourth-year classes in two different schools - one using the Dienes' method and the other using the Fletcher method - producing four groups of 16 subjects each. The groups

were matched for I.Q. and socio-economic status. Students had been in the same program for two and six years, respectively.

b. Independent Variables

The independent variables were the teaching methods. The Dienes' method employed blocks of different shape, size, and color which children manipulate. The Fletcher method was a text and workbook approach consisting of abstract paper-and-pencil exercises. Much of the text was in the form of diagrams and pictures.

c. Dependent Variables

The dependent variables were scores on tests of concept formation.

d. Tests

"Two sets of 16 cards were prepared each containing pictures of either two geometrical shapes (circle and triangle) or two common objects (cup and spoon)". The first set was classified as abstract stimuli, the second as real-life stimuli. Each set contained cards showing all possible combinations of shapes and color (black and white).

e. Procedure

After a trial experience, subjects were asked to figure out which kind of card the experimenter was thinking of. The cards were presented one by one by the subjects. The experimenter placed each either in a "yes" or "no" column. There were four possible "yes" combinations and 12 "no" for each task. Performance was measured by the number of cards selected before presenting all of the "yes" cards or before subjects verbalized the concepts. There were four types of problems -- color-color (CC), size-size (SS), color size (CS), size color (SC) -- for each set of cards. Each child was given one problem of each of the four types. "One child in each group received each of the 16 possible orders of the four problem types".

4. Findings.

The table below shows the mean total number of cards required to solve the four problems for each group.

Mean Number of Cards Needed to Solve Four Problems
in Each Condition

Age Groups	Dienes		Non-Dienes	
	Abstract concepts	Real-life concepts	Abstract concepts	Real-life concepts
8-9 years	31.06	26.69	30.44	30.13
10-11 years	25.00	21.19	19.56	23.06
Mean	28.03	23.94	25.00	26.59

A four-way analysis of variance was performed on observed data to test for significance of differences. Results of the analysis indicated non-support for the five hypotheses listed above. However, within the Dienes group real-life concepts were easier to attain than abstract concepts, while there was no difference between the two types within the Fletcher group. Other differences, such as the fact that older children do better than younger children and that more complex associations are more difficult to recognize, were found.

5. Interpretations

The authors concluded that the over-all results did not support the Dienes' claim for the concept formation advantages of "dynamic search activity", including the expected improvement with age.

The relatively better within-group performance of "the Dienes children" on concept formation with real-life stimuli was taken to suggest a slight superiority for the Dienes group in generalizing relevant reasoning skills to out-of-school situations.

The findings relating to I.Q. and sex were discussed in relation to other relevant studies.

Abstractor's Comments

Although this is an interesting study, one is impelled to express some skepticism as to the strength of the general conclusion. These concerns arise from apprehensions relating to such aspects of the study as the small sample size, the unit of analysis employed (the subject), the

matching procedure employed, and the post-hoc nature of the study which severely limited control over the independent variables.

The relatively better performance on real-life stimuli within the Dienes group is not easy to understand. It is hard to believe, especially for 10- to 11-year-old children, that it is relatively more difficult to transfer generalizations from triangular objects to pictures of triangles than from triangular objects to things such as cups and spoons. The suggestion offered is not easy to accept.

The study ought to be replicated with the instructional method more carefully controlled over a shorter period, if need be with a larger or random sample and/or more sophisticated control over non-experimental variables. The findings could be different.

Despite recent progress in the domain of "meta-analysis", this commentator still holds the view that more carefully controlled studies capable of replication offers the best hope of providing research findings that can better support instructional decision-making.

Cornelius, M. L. and Cockburn, D. INFLUENCES ON PUPIL PERFORMANCE.
Educational Research 21: 48-53; November 1978.

Abstract and comments prepared for I.M.E. by PAUL R. TRAFTON,
 National College of Education.

1. Purpose

The purpose of the study was to investigate the influence of home background, age, and sex on the academic performance of 16-year-old students in a British school district.

2. Rationale

It is well recognized today that a great number of factors have a bearing on student performance in school. In particular the role of environmental influences on educational achievement is receiving careful scrutiny by educational writers and researchers. The present report was part of a larger study to examine "possible handicapping conditions on pupils and schools and to attempt to discuss links between academic achievement and handicapping conditions."

3. Research Design and Procedures

The entire age 16-plus group of 1152 students in the Derwentside District of County Durham, England formed the subjects of this study. Students were categorized by home background (5 levels), by age (4 levels based on quarters of the year), and by sex.

Academic achievement was measured by performance on two well-recognized tests: the General Certificate of Education (GCE) O Level examination and the Certificate of Secondary Education (CSE) examination. Achievement levels for all students were reported, together with those of external examination entrants; that is, students who might be described as having a strong academic orientation and plans for post-secondary education.

4. Findings

- a. Home Background: The data show a consistent increase across levels of home background, from lowest to highest level, for

performance in all subjects, English, and mathematics for all students and the subset of examination entrants. The researchers report a highly significant difference between the five means for the "all subjects" performance, with the exception of the two lowest levels, as well as between means for the extreme groups in English ($p < .004$) and mathematics.

- b. Age: The effects of age were apparently not as great, although there was a steady decline from oldest (autumn-born) to youngest (summer-born) students. The greatest differences occurred among examination entrants, with highly significant differences reported between the means of autumn- and summer-born students.
- c. Sex: For overall performance in all subjects there was little difference between boys and girls. However, there were great achievement differences between sexes in mathematics where boys greatly outperformed girls ($p < .004$), and in English where the opposite results occurred. The authors report much stronger performance for boys in mathematics, science, history/geography, and technical subjects. Girls outperformed boys in the areas of English, languages, commercial education, religious education, and domestic science. Performance was about equal in the art/music area.

5. Interpretations

Several qualifications are noted, including the limitation of the means used to assess academic achievement, the problem of determining and assigning students to home background categories, and the fact that certain variables could not be measured nor controlled, such as teacher effects, influences of other students, and attendance patterns.

Two primary conclusions are drawn. First, there exists a strong relationship between academic achievement and home background. Not only did students from the highest levels do significantly better than those from the lowest level, but a greater percentage of them were entered for external examinations. Second, there are strong differences in achievement patterns for boys and girls, with boys achieving much better in

mathematics and science and the girls in English and language. The authors note, "Whilst the fact that differences exist is itself not surprising, what is perhaps unexpected is the actual size of the difference in the four subjects just quoted."

Abstractor's Comments

Within the limitations of this type of research, the study is well done and provides program evaluation information that is useful in planning more detailed research in which variables can be more carefully controlled and manipulated. It also provides further data to add to the growing body of information on the effects of certain factors on learning.

As useful as these studies are in terms of raising "red flags" with respect to program planning and further investigation, they also are a cause for some concern. This type of data is frequently seized upon by the public and some educators and used as evidence to "prove" some notion. There is the tendency to ignore the limitations of this type of study and the tentativeness of the findings. It also needs to be recognized that these studies are unable to provide any suggestion for why the findings occur or what can be done to remedy the problem. As the authors note, "The evidence ... exposes a number of problem areas without really being able to offer any solutions." Perhaps there is a greater need at present to place efforts into investigating these matters more intensively and investigating the effects of intervention programs that attempt to deal with the problems.

Finally, the study keeps before us the recurring problem of the achievement patterns of disadvantaged students and females in mathematics and reminds us of the need to deal with them directly.

Days, Harold C.; Wheatley, Grayson, H.; and Kulm, Gerald. PROBLEM STRUCTURE, COGNITIVE LEVEL AND PROBLEM-SOLVING PERFORMANCE. Journal for Research in Mathematics Education 10: 135-146; March 1979.

Abstract and comments prepared for I.M.E. by MARILYN J. ZWENG,
The University of Iowa.

1. Purpose

The study examined the differences between the problem-solving processes used by concrete-operational eighth graders and formal-operational eighth graders. The effect of problem structure, either "simple" or "complex", on the use of strategies was also investigated.

2. Rationale

Several studies dealing with the relationship between problem-solving processes and cognitive level have shown that there is a marked change in the use of problem-solving processes during the transition from concrete- to formal-operational thinking. The new information to be gained from this study was not explicitly stated. It appears, however, that prior studies examined problem-solving processes in a very general sense, whereas in this study the relationship between cognitive level and the use of specific processes such as understanding processes, recall processes, trial-and-error strategies, and so forth was of concern.

With respect to problem structure, prior studies have shown that the use of problem-solving strategies is affected by problem structure, difficulty, content, and context. The rationale for again examining the effect of problem structure is given by the authors when they note that earlier studies "tended to combine syntactical and structural variables under one category." "Consequently, no definitive conclusions can be reached from these studies about the effect problem structure has on problem-solving performance."

3. Research Design and Procedures

The subjects were 29 concrete-operational and 29 formal-operational eighth-grade students enrolled in general mathematics. The two groups were randomly selected from a larger population of students who had been previously classified as concrete- or formal-operational using a

paper-and-pencil test. Subjects were asked to solve four "simple" and four "complex" problems in an interview setting. Problem structure was classified as "simple" or "complex" using an instrument developed by Days. All problems could be "solved algebraically using one or more linear equations." However, this was not the solution technique that the eighth graders were expected to use. Subjects were directed to "think-aloud" while solving their problems. Interviews were tape-recorded. The problem-solving processes and strategies used by the subjects were determined from the tapes using a modified version of Kilpatrick's coding system. Fifteen processes, divided evenly into five categories and three strategies, were identified in the coding system. The five categories were (1) Understanding Processes, (2) Representational Processes, (3) Recall Processes, (4) Production Processes, and (5) Evaluation Processes. Although three strategies were listed in the coding system, results were reported only for the strategy called "systematic trial-and-error." Separate sets of scores were recorded for the four simple problems and the four complex problems. The following scores were tabulated:

- Process scores the number of problems in a set on which a subject used a given process category, as for example, the number of problems in the "simple" set on which the three "Understanding Processes" were used.
- Range scores the number of different processes and strategies used by a subject within each of the two problem sets.
- Difficulty score the proportion of problems within a set (simple or complex) that were solved correctly.

Analysis of variance for a 2x2 factorial design was used to analyze the range, understanding, production, and evaluation scores. Wilson's two-way ANOVA based on medians was used to analyze representation, recall, and systematic trial-and-error process scores.

4. Findings

Cognitive Level

1. Concrete and formal operational students did not differ with

respect to their user of understanding, representational, and recall processes. However, representational and recall processes were seldom used.

2. Formal subjects used production processes, evaluation processes, evaluation processes, and systematic trial-and-error significantly more often than concrete-operational subjects.
3. Range scores were significantly greater for formal students. That is, they used a wider variety of problem-solving processes and strategies than did concrete students.

Problem Structure

1. All problem-solving processes and strategies, except recall processes, were used more frequently on complex problems than on simple problems. "Recall" was almost never used.
2. The number of different processes used (the range) was also greater for complex problems than for simple problems.
3. There was no significant difference in difficulty between simple and complex problems for the concrete students. Scores of concrete students were very low for both sets. Only 34% of the simple problems and 22% of the complex problems were solved correctly.
4. For formal operational students, complex problems were considerably more difficult than the simple problems. The formal students got 84% of the simple and 59% of the complex problems correct.

Interactions

Interactions between structure and cognitive level were reported for production and evaluation processes. In both cases formal students used the processes more frequently on complex problems. For concrete students, however, problem structure did not significantly affect their uses of the two processes.

5. Interpretations

The authors point out the following limitations of the study: the two cognitive groups may not have been entirely distinct, the set of problems may have contained too few problems to detect small differences in performance related to problem structure, and the "think-aloud" tech-

nique provided only partial information about the processes students used. The authors' interpretation of the results is contained in the following quotations: "The fact that concrete and formal subjects for the most part did not differ in process use on the simple structure problem suggests that simple structure problems may not have evoked the use of "high level" processes. If this was the case, then many textbook problems probably fail to elicit the use of "high level" processes, also. The latter statement is based on the fact that the simple structure problems were typical of many of the problems found in seventh- and eighth-grade mathematics textbooks." ... "In any case, it seems quite clear that both task variables (problem structure) and subject variables should be given proper consideration when studying or teaching problem solving."

Abstractor's Comments

The results indicating that formal-operational students solve problems more successfully and use a wider variety of processes than concrete students are certainly not surprising. This is precisely what is implied by the definition of "formal-operational." The results simply confirm that the students had been assigned to the correct cognitive levels initially. The result, which implies that more processes are used to solve complex problems than simple problems, is also no surprise. Although very little information was provided about the criteria for classifying problems, it is probably safe to assume that complex problems were those which required the use of more processes; therefore, if they were solved, more processes would have to be used to solve them.

Those results that were "interesting" were the ones which dealt with specific processes. Unfortunately, much information about process use was omitted in the report. We are told that production and evaluation processes were used more frequently by formal students than concrete students, but within the production category there were three processes: reasons deductively, uses successive approximations, and estimates. Within the evaluation category the three processes were "checks manipulations," "checks conditions," and "checks by retracing steps." The processes within each category are quite different. Were they used equally, or was one of the processes used more frequently than others?

The answer to this question could have implications for instruction. The authors avoid instructional issues by arguing that "The effective use of production and evaluation processes ... may have required formal thought." But one could argue on the other hand that if, for example, "successive approximations" was a technique frequently used by successful problem solvers, perhaps less-successful problem solvers could be taught to use successive approximations. (The abstractor has intentionally equated "successful problem-solver" and "formal-operational".) Studies on teaching general heuristics have been notably unsuccessful, but teaching a specific heuristic might be successful. This report could have provided some guidance about the specific heuristics which might have potential for future investigations.

When assessing the results on process use, the reader is also limited by not having access to the problems. Only two of the eight problems were provided in the report. Different problems may evoke different processes and consequently the nature of the problems might have explained some of the results. The classification of problems into the two "structures", simple and complex, appears to have been based on the complexity of the equations which would be used to solve the problems if they were solved algebraically. This classification system, however, does not account at all for other structural variables that could have a significant impact on process use. For example, problems about perimeter or areas of rectangles would very likely generate the use of representational processes (which were used very little by the subjects in the study). To illustrate this concern for "other" structural variables, consider the two sample problems which were provided in the report:

Simple Problem. A cow and pig together cost 56 dollars. The cow costs 30 dollars more than the pig. How much does each cost?

Complex Problem. Jeff bought 5 oranges and 10 apples for \$1.65. How much does one apple cost? How much does one orange cost?

Both of these problems involve two "unknowns." Having two "unknowns" would seem to be a structural variable that could affect problem process. Since the students had not been taught algebraic techniques, it is fairly safe to hypothesize that this problem structure would evoke "suc-

cessive approximations," "checking conditions," and "systematic trial-and-error" processes which appear to have been used frequently. Other problem structures are not as likely to evoke these three processes. Some researchers are now beginning to recognize that there are probably "domain-specific" problem-solving processes where "domain" is meant to suggest a large group of problem types which have some structural features (other than simple or complex) in common. If studies on problem solving are going to provide some useful insights into improving instruction, researchers need to arrive at some common understandings of the meaning of "structure" and begin asking better questions about the relationship between structure and process.

It is clear from the preceeding comments that this abstractor's major concern when reviewing research on problem solving is, "What does this study say about helping kids learn to solve problems?" The authors alluded to this question by implying that if textbooks included more "complex" problems the use of problem-solving processes would improve. But what about the concrete-operational students who solved only 34% of the simple problems? Having more complex problems in the textbook isn't going to help them. Apparently the writers are assuming that the poor problem solvers couldn't solve problems because they weren't formal-operational. Perhaps it's the other way around. Was there nothing to be learned about how formal-operational students solved problems that might help these youngsters? The authors' second answer to the instructional question was: "... task variables and subject variables should be given proper consideration when studying or teaching problem solving." But what's "proper"?

The study was well-designed and the results are undoubtedly valid for the questions investigated. Furthermore, some of the issues which have been raised in this commentary are probably addressed in the dissertation from which the article is drawn. It would be dangerous, however, to interpret the results of this study to mean that certain kids simply can't solve problems and only maturation will help them. We are abdicating our responsibilities as mathematics educators if we don't look for ways of using the instructional process to improve problem-solving strategies and performance.

A NOTE OF CLARIFICATION in regard to:

Engelhardt, J. M. An Analysis of Children's Computational Errors:
A Qualitative Approach. British Journal of Educational
Psychology 47: 149-154; June 1977.

This article was abstracted in IME in Volume 12, No. 1, Winter 1979,
pages 24-26.

Jon Engelhardt provided the following comments:

In my recent article abstracted in IME the reviewer noted what he interpreted to be a computational error with respect to the base number used to calculate percentages for the various error types. Earlier in the article it was stated that:

... it was accepted that such errors might be classified as more than one error type. Although this restriction may have resulted in non-mutually exclusive error types, it was decided that realistic error descriptors were more important than statistical neatness (p. 150).

As correctly summed by the reviewer, this resulted in 2,687 errors classified from 2,279 incorrect items. For purposes of the percentages reported in Table 2 and referred to elsewhere in the article, the total number of incorrect items (i.e., 2,279) was used as the base, rather than the number of errors classified.

An earlier version of the manuscript had noted this fact. However, this was somehow omitted in the editing/revision process. The difference in base number affects little the results of the study. It is hoped that those interested in the specific percentages will note this difference.

The review also noted disagreement with the statement that little research has focused on the "qualitative aspects of children's errors." While the reviewer is obviously entitled to his opinion, even a brief review of the literature suggests that, while a dearth of investigations on mathematics achievement exists, relatively few have studied computational errors as their main subject of inquiry. Furthermore, as found in many of those studies, it seems that error classifications like "regrouped from the tens column when it was unnecessary" adds little to our understanding of children's mathematics learning. Although the reviewed study suffers to a

degree from that criticism, the kinds of classifications made seem more descriptive in ways that refer to conceptual understanding (or misunderstanding), rather than pure procedure.

To provide even more valuable information on children's mathematics learning, the emergence of and changes in children's computational errors (or erroneous approaches) over time needs to be examined. Such an examination may ultimately suggest errors as symptomatic of general mathematics learning (or misconceptualization).

Good, Thomas L. THE MISSOURI MATHEMATICS EFFECTIVENESS PROJECT: A PROGRAM OF NATURALISTIC AND EXPERIMENTAL RESEARCH. Paper presented at 1978 annual meeting of AERA. ERIC: ED 159 056.

Abstract and comments prepared for I.M.E. by JAMES T. FEY, University of Maryland.

1. Purpose

Two purposes were indicated:

- a. To determine whether teachers of elementary school mathematics could be trained to use classroom instructional patterns that previous research had shown to characterize more effective teachers.
- b. To determine whether teachers who adopted the new system of instruction were more effective than a control group receiving no special guidance.

2. Rationale

In earlier phases of a research program to which this study contributes, Good and Grouws (1975) identified factors in the classroom instructional programs of elementary mathematics classes that distinguished consistently effective from consistently ineffective teachers. They found that lessons of effective teachers contained the following components:

- Daily Review ($\approx$ 8 minutes)
- Development, with special attention to establishing clear understanding ($\approx$ 20 minutes)
- Seatwork ($\approx$ 15 minutes)
- Homework ($\approx$ 15 minutes per day, except Friday)
- Special Reviews (weekly and monthly)

The problem is to induce elementary teachers to implement these guidelines--most of which fit recommendations long urged by 'experts' in the field--and to determine whether the resulting program of instruction would be more effective.

3. Research Design and Procedures

In Fall 1977, 40 fourth-grade teachers in 27 semi-departmentalized Tulsa elementary schools--most low income, Title I schools--volunteered

to participate in a four-month experiment. Half were assigned to a "delayed feedback" treatment, told they would get details of the project in February, and directed to teach as they usually would in the meantime.

The other teachers were given an initial 90-minute training session introducing the proposed experimental teaching program. They were urged to read a 45-page manual explaining the treatment, and they met again two weeks later for another 90-minute session to clarify the proposed instructional system. The period of experimental teaching was October 3, 1977 to January 25, 1978.

Data collected included:

- a. Observations of all 40 teachers (6 times each) using both high and low inference methods.
- b. Student achievement scores on SRA, Short Form E, Blue Level test (late September and mid-December) and a special mathematics content test designed to focus on material covered during the experiment.
- c. Measures of student learning styles and preferences and attitudes toward mathematics (September and January).

4. Findings

Data from the classroom observations showed that experimental teachers did, to a great degree, adopt the instructional model proposed for their use. Control teachers made much less use of the factors considered to be effective, such as review, checking homework, mental arithmetic practice, or active involvement of students in seatwork.

Achievement data showed that students in classes of experimental teachers significantly outperformed those in control classes.

Student Achievement on SRA Test		
	Pre-Test Raw Score	Post-Test Raw Score
Experimental	11.75	19.94
Control	12.61	17.60

[Since the preliminary report abstracted here, more detailed analyses have been carried out and reports given in Ebmeier and Good (1979) and Good (1979).]

5. Interpretations

Based on this initial data analysis, the author concluded (a) that the basic instructional program had been demonstrated to be easily and effectively put into practice, and (b) that the key factors identified by earlier post hoc description of effective teachers showed strong promise of working to help other improve their effectiveness.

Abstractor's Comments

The design and procedures of this study are good evidence that the methodology of research on instruction has made striking progress in the past 10 years. The investigator has taken the time and energy to (1) involve a large (for teaching experiments) number of classes, (2) measure carefully to see that the intended treatments actually occurred, and (3) build a program of research moving cautiously from description to experimentation. Furthermore, the key variables and treatment implementation methods are simple enough to offer real promise of influencing what happens in schools--not complex and subtle factors that a busy classroom teacher has little hope of utilizing.

Looking at the description of the "experimental" instructional program (elaborating on the five components listed above in the Rationale section), one could easily argue that the study shows simply "If teachers try to do what experts have long been urging, they will be more effective." The preliminary analyses reported in this paper do not shed much light on the relative importance of each component in the program. Is review, homework, development, or seatwork most important? Further, it is natural to wonder if the influential factors are the same when the student achievement measure is changed--to emphasize understanding or problem-solving processes, for instance. The investigators made note of these issues and plan to address several in subsequent analyses.

References

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- Ebmeier, Howard and Good, Thomas L. The Effects of Instructing Teachers About Good Teaching on the Mathematics Achievement of Fourth Grade Students. American Educational Research Journal 16: 1-16; 1979.
- Good, T. L. Teacher Effectiveness in the Elementary School. Journal of Teacher Education 30: 52-64; 1979.

Hendrickson, A. Dean. AN INVENTORY OF MATHEMATICAL THINKING DONE BY INCOMING FIRST-GRADE CHILDREN. Journal for Research in Mathematics Education 10: 7-23; January 1979.

Abstract and comments prepared for I.M.E. by CAROL A. THORNTON, Illinois State University.

1. Purpose

To make an inventory of the responses of first graders to simple situations involving counting, number, computation, and place value,

2. Rationale

The kind of mathematical thinking that beginning first graders are capable of, if they have manipulative materials to support their thinking, is of interest to teachers and curriculum developers who design, sequence, or carry out learning activities for this age group. Common practice in use of school texts relies heavily on children's ability mentally to interpret or manipulate pictured objects and symbols. Studies of children's interpretation of pictures and mathematical symbols are few. Investigations by the Project for the Mathematical Development of Children (Behr, Erlwanger, and Nichols, 1974; Hamrick, 1975) and one conducted by Campbell (1975) indicate that children often interpret symbols or pictures in textbooks in ways different from those the author had in mind. Other earlier studies analyzed the relative effectiveness of manipulative, pictorial, symbolic, and verbal approaches; e.g., Neufeld and Scott (1976), Grunau (1976), Johnson and Steffe (1971), Brownell (1941). Such studies do not give clear direction to curriculum developers who must help young children conceptualize number and basic operations with number. The need was felt to analyze more closely children's thinking in oral quantitative situations where the use of materials, though optional, was encouraged.

3. Research Design and Procedures

The sample consisted of five girls and five boys randomly selected from the first-grade classes of six elementary schools. The schools, with a total first-grade enrollment of 314, were chosen to represent a

broad range of socioeconomic backgrounds. All 30 boys and 27 girls were interviewed individually. (Due to absences, three girls were not interviewed.) Each child was presented with 17 tasks related to counting, number, computation, and place value. All questions posed were repeated, and key words were repeated whenever necessary to clarify a situation for a child. Base ten longs and units were made available. If a child did not spontaneously use the materials, a suggestion to do so was made. Interviews were taped and ranged from 20 to 30 minutes in duration. All interviews were conducted by the researcher within a three-week period during the first month of school. The interviews were later analyzed for observable trends and noticeable peculiarities. The written summary was divided into two parts: (1) Counting and Number Recognition (tasks 1 through 5), and (2) Using Addition, Subtraction, Multiplication, and Division (tasks 6 through 17).

4. Findings

For Part 1 (Counting and Number Recognition):

- a. Oral counting from 1: 37 children counted into the 40s before being stopped.
- b. Oral counting from 20: 40 children counted into the 40s before being stopped.
- c. The results of the next three consecutive similar tasks are summarized as follows:
 - 1) When five objects were arranged into one part of two and one part of three, 44 children gave the number of one of the parts rather than the whole.
 - 2) When twelve objects were partitioned into two parts, only nine failed to look at the whole rather than at one or the other of the two parts.
 - 3) When eighteen objects were partitioned into two groups, only four children were still giving the number of only a part.

For Part 2 (Using Addition, Subtraction, Multiplication, and Division):

- a. Addition situation: Starting with 2 blocks, 29 children correctly told how many 7 more would make (80 percent did not use extra objects to help compute).

- b. Addition situation: Only 11 correctly totaled 8 and 13.
- c. Subtraction situation: 46 could start with 8 objects and tell how many were left after 5 were given away (59 percent used objects to help compute).
- d. Subtraction situation: In similar situations, 52 correctly subtracted 3 from 8 (54 percent used objects); 49 subtracted 6 from 14 (88 percent used objects); and 56 could subtract 8 from 14 (79 percent used objects).
- e. Place value: Given 1 long and 2 units, and asked to "lend 6 little blocks" (units), 24 children made a correct trade of 1 ten for 10 units to carry out the request.
- f. Multiplication situation: 3 children could correctly tell the total for two groups of 4 in an oral problem setting (18 percent used objects); 14 children could tell the total of three groups of 5 in a similar setting (71 percent used objects).
- g. Division situation: 28 children could divide twelve into groups of 4 and tell the number of groups (79 percent used objects); 27 children could divide 15 into three equal-sized groups (89 percent used objects).
- h. Money: Only 11 children could tell how many nickels they would get for 23 pennies.
- i. Division situation: 24 children correctly told the number each would get when 26 marbles were shared equally with "three friends and yourself." All who answered correctly used objects to compute.
- j. Place value: Given 13 units and shown 3 tens, 35 children were willing to trade 10 units for 1 of the tens, convinced that after the trade they would still have 13.

5. Interpretations

Entering first-grade children show variations in counting ability and style. Their ability to count objects falls off rapidly as the

number of objects becomes greater than 15 or so. In particular, the ability to count orally does not guarantee the ability to count rationally. In considering situations in which a group consists of two or more identifiable parts, a large number of these children tended to compare the parts with each other or name the larger part rather than the whole.

Many entering first-grade children can use manipulative materials to represent the objects in orally stated problems; they can form groups, compare them, join them, separate them, and decompose them, as needed, to solve problems involving all four arithmetic operations.

Many entering first graders are more comfortable exchanging 10 units for 1 ten than vice versa (1 ten for 10 units). Many entering first graders seem able to handle situations that require the joining of a number of equal-sized groups. Some by their comments indicated a "repeated addition" kind of thinking; others clearly did not. Many entering first graders may not understand money relationships, an understanding sometimes assumed by text writers and teachers.

Four recommendations are made:

1. Preschool and first-grade children should have much experience with counting objects--particularly larger numbers of objects.
2. First-grade children should have experience with groups of objects that are separated into two and three or more parts. Recognizing that sets can be composed of many parts would help their understanding of part-whole relationships and establish a firmer basis for later work with number in arithmetic.
3. First-grade children should have the opportunity to use manipulatives to represent objects in real-world problems in the various comparing, joining, separating, grouping, and distributing operations that are related to adding, subtracting, multiplying, and dividing.
4. When presented with problem situations, children should often be asked to describe what they think they are being asked to do and what key words, particularly comparatives, mean to them.

Abstractor's Comments

Status studies like this one are critical to the development and planning of curricular programs in school mathematics. In particular, the present study extended earlier research efforts by focusing on the kind of mathematical thinking entering first graders are capable of when they have manipulatives to support their thinking. Looking at oral problem situations related to the four arithmetical operations for whole numbers, together with counting and place value, has produced four recommendations (cited above). Within the context of the study, these recommendations are both reasonable and practically significant. They flow from a study that was generally well-conceived and carefully designed and implemented.

In retrospect, it might be asked whether too many content topic areas were investigated. One wonders whether a greater contribution might not be made by pursuing one or two selected topics rather than a broad spectrum, as in the present study. Perhaps a suggestion for further research, an extension of the present investigation, is to apply a similar research design to the analysis of isolated topics. The study, as reported, is sufficiently clear and detailed enough to allow this modified replication. Another suggestion might be to pursue replication using materials more familiar to very young children. Only base ten units and longs were made available to children in the present study.

The one drawback to the journal report was the overstatement of conclusions in the discussion section. While an effort was made by the researcher to draw a sample from a wide range in socioeconomic and residential backgrounds, one wonders whether the results can be applied legitimately to all first graders. Perhaps a statement limiting interpretation of results to the population of the study would have been more appropriate.

The emphasis in the study on the techniques of oral problem solving in conjunction with the use of manipulatives has implications for future research, for curriculum development, and for the teaching of mathematics which, if anything, were a bit understated. It would be interesting to see more of a follow-up on this point both in the research literature and in the classroom.

Horak, Virginia M. and Zweng, Marilyn J. THE EFFECTS OF INDUCTIVE-DEDUCTIVE TEACHING METHODS AND FIELD-DEPENDENCE-INDEPENDENCE COGNITIVE STYLE UPON STUDENT ACHIEVEMENT IN MATHEMATICS. Paper presented at the Annual Meeting of the National Council of Teachers of Mathematics, San Diego, April 1978. ERIC: ED 161 700.

Abstract and comments prepared for I.M.E. by HELEN ADI, Northern Illinois University.

1. Purpose

This study examined (a) the main effects of two instructional treatments, the inductive and deductive teaching methods; and (b) the interaction effects of these instructional treatments with the cognitive style of field dependence/independence on the achievement measures of a group of college elementary education majors.

2. Rationale

This was an Aptitude-Treatment Interaction (ATI) study. It was based on the premise that educators must examine and adopt instructional treatments to student individual differences. The treatment dimension consisted of inductive and deductive teaching methods, and the aptitude dimension consisted of the cognitive style of field dependence/independence.

Research on inductive versus deductive teaching methods had produced contradictory results. In the present study, inductive teaching was defined "as being based on the presentation to the learner of a sufficient number of specific examples to enable him or her to arrive at a definite rule, principle, or fact." It was not considered "a necessary part of inductive teaching to state the generalization for the student, or to have the student state it after exposure to the examples." Deductive teaching was defined as a "method that proceeds from rules or generalizations to examples and subsequently to conclusions or to the application of the generalizations."

The work of Witkin and his colleagues was considered in choosing the cognitive style of field dependence/independence, the aptitude dimension of this study. Field-independent persons are better able to

structure their experiences and "to impose an organization upon ambiguous stimuli materials" than the field dependent persons. Further, field-independent persons perceive their environment analytically, while field-dependent persons tend to perceive their environment more globally. Thus, an interaction effect was expected to exist between inductive and deductive teaching methods and the cognitive style of field dependence/independence when achievement scores were considered as the dependent measures.

3. Research Design and Procedures

The subjects for this study were 118 college students enrolled in two sections of a Modern Elementary Mathematics course. The two sections were randomly assigned to either the inductive ($n=47$) or the deductive ($n=71$) teaching method. Both treatments included the same introductory materials and basic definitions of a unit on transformational geometry.

The deductive treatment followed a rule-example paradigm, and the inductive treatment followed an example-rule paradigm. For the deductive treatment, a rule or a principle was stated for the students, and then examples were solved. For the inductive treatment, examples were solved by the teacher and the students, and then students were encouraged, via questions, to formulate a principle of their own. A total of two hours and 30 minutes of instruction, within a two-week period, was delivered to both experimental groups.

An achievement test was administered to all students. The items of this test were grouped into knowledge, application, and analysis items based on Bloom's (1956) taxonomy of educational objectives. This classification was conducted by the investigator in agreement with others who were consulted. The application and analysis subtests were further grouped together and were referred to as the transfer subtest. The achievement scores defined the dependent measures. Witkin's (1954) Group Embedded Figures Test (GEFT) was administered to all students to measure field dependence/independence. Twenty minutes were needed to administer the GEFT.

4. Findings

One-way ANOVAS tested the main effects of the instructional treatments on five dependent measures: overall achievement, knowledge subtest, application subtest, analysis subtest, and transfer subtest. Significant treatment effects (at the .05 level) existed only on the mean performances of the transfer and the analysis subtests, favoring the inductive treatment. Pairwise comparisons of mean performances favored the inductive treatment for all dependent measures, but only for the analysis and transfer subtests were the differences significant.

Pearson product moment correlation coefficients between the achievement measures and the GEFT scores were computed separately for each treatment group. Within the inductive group the correlation coefficients ranged from .09 to .29. Within the deductive group the correlation coefficients ranged from .37 to .54.

Linear regression analyses tested the interaction effects between the instructional treatments and field dependence/independence on the achievement measures. Significant aptitude-treatment interactions ($.05 < p < .10$) for overall achievement and transfer measures were found. The Johnson-Neyman technique was utilized to provide information on the regions of significant differences for these interactions. When overall achievement was the dependent measure, and at a .05 level of significance, it was found that all students having GEFT scores of 5 or below out of a possible 18 performed better if they were taught by the inductive method. For the other students the instructional treatment did not result in a significant difference in achievement. When the transfer scores were the dependent measure, students having a GEFT score of 9 or below performed better if they were taught by the inductive method; otherwise, there were no significant effects of treatment on achievement.

5. Interpretations

Results of this study did not support the conjectures of Witkin et al. (1962) and Messick (1970) that field-dependent students learn better under a deductive teaching method while field-independent students learn better under an inductive teaching method. The authors interpreted the results of this study in light of the social interaction factor.

Previous studies had reported that field-dependent students were more socially inclined than field-independent persons. Thus, "it could have been that the field-dependent students in this study achieved more with the inductive method, due to the social interaction it afforded them." The inductive treatment did not force students to interact socially during problem solving but did allow it, whereas very little social interaction occurred during the deductive treatment.

Abstractor's Comments

The conceptual framework of this study is of interest to many mathematics educators. It may be quite difficult to "adopt instructional treatments to individual differences" in actual classroom situations; however, results of ATI studies may at least increase our awareness that instructional treatments may optimize achievement levels differentially depending upon the students' aptitudes.

- (1) The rationale of this study was very clear, and the authors were careful in defining what was meant by inductive and deductive teaching methods. Witkin's (1962) definition of field dependence/independence was adopted. No reference was made to more recent definitions of this cognitive style (e.g., Pascual-Leone, 1974; Witkin, 1977).
- (2) The sample consisted of two sections of college students who were enrolled in a Modern Elementary Mathematics course. The two sections were randomly assigned to either the deductive or the inductive instructional treatment. Sections and not subjects were randomly assigned to treatments. Since subjects were not randomly assigned to treatments, each subject's performance cannot be considered as a unit of analysis for ANOVA. Thus, the sample size of this study was $n = 2$ sections, and not $n = 118$ students as was stated.
- (3) Achievement tests were constructed by the authors to assess different levels of knowledge on the unit of instruction. No reliability measures on these tests were given. The GEFT was administered to all students. What was the reliability measure of the performance on the GEFT?

- (4) Since class sections, and not subjects, were the units of analyses, it was inappropriate to conduct ANOVA's with a sample size of $n=2$. Further, mean performance scores on the achievement tests were not reported. It is not sufficient just to report that "the mean for the inductive group was higher than that of the deductive group."
- (5) The authors were not specific enough in reporting their results. For example, the following appeared in the Results section:

... An informal check was made on the assumption that the Y-scores were normally distributed about the subpopulation means. This was done by examining the cumulative frequency distributions of the treatment groups for each of the criterion measures. The distributions appeared to be negatively skewed, although not enough to affect the results based upon the use of the F-statistic.

Nowhere in the article did the authors present the "Y-scores," the "frequency distributions," or the "subpopulation means" to which they referred.

- (6) The authors claimed that "the regression analyses of the data indicated strong support ($.05 < p < .10$) for the existence of aptitude-treatment interactions between field dependence/independence and the instructional procedures for the criterion measures of overall achievement and transfer." No F-values testing for interactions with field dependence/independence were reported. Was the reader expected to predict the F-values and to test for their statistical significance? Or was it that the reader was expected to believe that ($.05 < p < .10$) was an indicator of "strong support" for significant interactions? In general, the results were poorly reported.
- (7) Interesting conclusions were drawn out of the data although some of the statistical tests were inappropriately used. Field-dependent students learned more from the inductive method when overall achievement and transfer were measured, whereas field-independent students learned equally as well from the inductive and the deductive methods. This finding was interpreted in terms of the social interaction factor. Field-dependent students are socially inclined individuals, and the inductive treatment provided them with more

instances for social interaction than the deductive treatment. McLeod and Adams (1979) investigated the effect of social interaction on the learning of field-dependent students in inductive instruction. They reported that the different patterns of social interaction in the individual versus the small-group treatments did not appear to be related to the significant ATI's that were identified. Instead, they identified the different levels of guidance obtained from the teacher to affect differentially the performance measures of field-dependent students.

- (8) With all the contradictory results reported in the related ATI studies, the following question may be raised: "Would the authors have arrived at the same results had they replicated their study?"

References

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- Witkin, H. A. and Goodenough, D. R. Field Dependence Revisited. (ETS Research Bulletin 77-16). Princeton, N.J.: Educational Testing Service, December 1977.

Johnson, David C. and Harding, Robert D. UNIVERSITY LEVEL COMPUTING AND MATHEMATICAL PROBLEM-SOLVING ABILITY. Journal for Research in Mathematics Education 10: 37-55; January 1979.

Abstract and comments prepared for I.M.E. by NORMAN L. WEBB, University of Wisconsin-Madison.

1. Purpose

This study investigated if participation in computing courses contributes to some general overall mathematical problem-solving ability.

2. Rationale

As an outgrowth of the Cambridge University Computer-Aided Teaching of Applied Mathematics (CATAM) project, initiated at Cambridge, England in 1968, two computing courses were developed. The aim of the project was to improve the teaching in applied mathematics and theoretical physics for the Mathematical Tripos (three annual examinations called Part IA, Part IB, and Part II) by the use of computers in lecture courses and classes. The two computational mathematics courses (CATAM) were offered on an elective basis in the second and third years of the Mathematical Tripos. The second-year course consisted of 16 one-hour lectures and 8 two-hour practical sessions. The third-year course consisted of 16 lectures and 16 practical sessions. The students in these courses wrote and ran programs with the potential for rapid interaction and optional graphical display of results (intermediate or final).

The faculty sets the examinations with a concerted attempt to make the questions open-ended problems. The Part IB examination consists of six three-hour papers (16 questions for each paper). The Part II examination is similar, with the exception that candidates have an opportunity to choose among more courses. Marks for the computational mathematics courses are awarded on the basis of written reports different from the open-ended problems for the normal Tripos examination. The computing marks are added to the written paper totals after being scaled in such a way that the maximum possible score is less than the maximum that could be obtained by answering correctly all questions for any other 16-lecture course.

3. Research Design and Procedures

Certain basic assumptions were made in conducting this study. First, the Tripos examinations were used as measures of problem-solving ability. The Tripos IA (year 1) was used as the pretest or independent measure. Tripos IB and Tripos II were used as the dependent measures. Second, the performance on Tripos IB or Tripos II was assumed to be linearly related to Tripos IA. This assumption was made even though each examination is used in selecting who is eligible to take the next examination. Third, differences in posttest performance when adjusted for any initial differences in ability was assumed to be attributed to the computing course. It was noted that no students had the opportunity for such experiences during pre-university education.

An "evaluation model" was used to analyze the data from four classes of students (1971-73, 1972-74, 1973-75, and 1974-76) for each of three years (Tripos scores IA, IB and II). The data were recorded for three subgroups of students (pure, applied, and mixed) based on subsections of the examination taken both with and without the computing score for Tripos IB and II. These scores, first without, then with, the computing scores, were then used in the analyses.

To analyze Tripos IB (year 2) given Tripos IA, $4 \times 3 \times 2$ (class by subgroup by computing course or no computing course) design was used. To analyze Tripos II (year 3) given IA or IB, a $4 \times 3 \times 3$ (class by subgroup by third-year computing course or second-year computing course only or neither computing course) design was used. In each analysis, longitudinal data that were available on the different groups of students were used. The number of students in each of the four classes that took all three examinations were 193, 213, 224, and 207, respectively. Computing and non-computing groups were compared first on examination scores without any computing scores. This gave the non-computing students an advantage. Then comparisons were made with the computing scores included.

A series of regression of analyses was done to answer the question of whether using individual regression equations (for cells or columns) rather than the equation for grouped data (rows or total) significantly reduces the sum squares about the regression equations. Then the derived regression equations were used to calculate predicted scores for

subgroups, with 50 percent confidence intervals. If the confidence intervals did not overlap, the subgroup with the higher predicted score was considered as superior in achievement. The three subgroups used were those who performed at the mean on the pretest variable (Tripos IA), those who performed at one standard deviation above, and those who performed at one standard deviation below the mean. Fifty percent confidence intervals were selected since a primary purpose of the investigation was to identify trends.

4. Findings

The F ratios for the dependent variables IB (given IA) and II (given IA) for the conservative measures, when computing scores were excluded, failed to reach significance (only 1 of 31 ratios was significant at $p \leq .05$). When the computing scores were included, enough of the F ratios were significant (15 of 31 ratios at $p \leq .05$) to proceed to calculate the confidence intervals and compare different groups of students. In this analysis for Tripos IB and Tripos II without including computing scores (the conservative approach), most comparisons favored computing (19 out of 48) or indicated no difference (27 out of 48). When computing scores were included, the comparisons consistently favored computing students (33 out of 36).

These results were validated by using Tripos IB (year 2) as the independent variable for the analyses of Tripos II where the number of comparisons favoring each group was consistent with the first analyses using Tripos IA. No consistent pattern was found to suggest that the computing activity favored one particular type of student (pure, applied, or mixed; or high, middle, or low within these classifications for IB/IA and II/IA).

5. Interpretations

The computing courses were concluded to have made a significant contribution to performance on the Mathematical Tripos examination. Since the students at Cambridge University are of a high ability, such a computing experience was viewed as being highly beneficial for the mathematically talented.

Abstractor's Comments

Certain basic assumptions were made in this study that are critical in interpreting the results:

- (1) The examinations were accepted as a measure of problem-solving ability.
- (2) Any differences in examination scores among computing and non-computing students were the result of the computing courses.

Although the authors were very clear in pointing out these assumptions, the validity of the assumptions are highly in doubt. The selection process of students attending Cambridge University ensures that the population investigated is atypical and is most likely a group of highly successful problem solvers. The variation in scores on the examinations then is probably related to the knowledge base acquired in the mathematical areas studied rather than general problem-solving ability.

It is questionable that the computing courses were the only cause in the differences in scores. Other factors that may affect the differences are the initial differences between students who selected the computing courses and those who did not; content taught in the other elective courses; and differences in other courses taken by the computing and non-computing students. There are some obvious differences between the students selecting the computing course the second year and the other students on the mean scores on Tripos IA (first-year examination). The computing students had a higher mean score for each class of students.

Without more information about what goes on in the computing courses and how this compares to other courses, it is very difficult to reach the conclusion that the computing courses made a significant contribution to overall mathematical problem-solving ability. Any differences between groups on the examination scores are also confounded by having computing scores included for the computing students. Students were not compared on the same criteria.

Schoenfeld, Alan. PRESENTING A STRATEGY FOR INDEFINITE INTEGRATION.
American Mathematical Monthly 85: 673-678; October 1978.

Abstract and comments prepared for I.M.E. by RICHARD CROUSE,
 University of Delaware.

1. Purpose

The purpose of this study was to determine if students' abilities to evaluate integrals can be enhanced by providing them with a coherent, problem-solving approach.

2. Rationale

Students in a first-year calculus class should, with a reasonable amount of practice, be able to evaluate indefinite integrals at a very high performance level. Yet they have more difficulty evaluating integrals than they should in spite of many hours of working on practice problems. The author argues (1) that these difficulties occur largely because students lack a coherent and efficient means of approaching problems in integration, and not because they have difficulty learning the important techniques; and (2) that students' abilities to evaluate integrals can be enhanced, with no extra study time, by providing them with a coherent approach.

3. Research Design and Procedures

Instructional material consisting of a workbook text, Integration: Getting It All Together, and a solutions manual were devised based upon the following methodology:

- a) Determine, as accurately as possible, what your experimental audience can and cannot do.
- b) Examine, in minute detail, the action of "experts" as they perform the desired task. The purposes of this examination are to: (1) discover regularities, strategies, or implicit rules the "experts" may be using - often without being conscious of them - as they solve problems; and (2) decompose the problem-solving process into smaller, more manageable "chunks" which can be taught individually and then carefully combined.
- c) Create a model of the problem-solving process based on the

chunks discovered in (2) making explicit use of the strategies discovered in (1). Design instructional materials which make both accessible to the students.

- d) Test and revise both the model and the instructional materials.

In particular, the workbook text was divided into three major stages: (1) simplify the integrand; (2) classify the integrand, i.e. use the form of the integrand to indicate the appropriate method of solution; (3) modify the integrand.

The instructor of a second-quarter calculus class agreed, four days before an examination, to distribute the materials to a randomly chosen half of his class. Two students were absent the day he handed the materials out, so eleven of twenty-six students actually received them. These eleven students were asked to prepare for the test using the materials, rather than by solving the miscellaneous exercises in their text. No one was available to clarify the strategy for the students in the experimental group, so whatever they learned would be from the materials alone.

The subjects were tested on an examination constructed by the instructor of the course and consisted of nine questions. Seven of the questions on the examination were indefinite integrals, while two questions dealt with theoretical issues related to integration.

4. Findings

- a) The experimental group outscored the control group on six of seven questions ($p < 0.01$). The materials did enhance student's ability to evaluate indefinite integrals.
- b) The experimental group indicated that they averaged (during a four-day period) a total of 7.1 hours each, while the control group indicated that they averaged a total of 8.8 hours each.

5. Interpretations

The investigator concluded from his results that:

- a) Most students can learn to perform each of the individual techniques of integration reliably, as long as they know which technique they should be using.
- b) Students often find it very difficult to select appropriate approaches to problems encountered out of context.

- c) We cannot expect most students to develop coherent strategies for approaching problems if they are left on their own to do so.
- d) With careful observation of "expert" behavior, it may be possible to delineate a strategy which, under ideal conditions, produces problem-solving behavior comparable to that of the experts.
- e) Such a strategy, if properly presented to students, can have them solving problems more effectively after less total study time.
- f) Problem-solvers must have a mastery of the techniques that are commonly used in that area.
- g) Problem-solvers must have a strategy that enables them to determine quickly and accurately which techniques they will bring to bear on any particular problem.

Abstractor's Comments

Among the questions which arise in connection with the reporting of this study are:

- a) The study was conducted on quite a small sample.
- b) Was the Hawthorne effect a confounding factor for the experimental group?
- c) Reliability and validity measures were not given on the test taken by the subjects.
- d) The non-experimental design is a serious weakness, but it was dictated by the available instructional setting.
- e) Was the appropriate statistical test used or should an appropriate t-test have been conducted?

In spite of these criticisms, this is an interesting, clearly written study which attacks a very important problem in the teaching of mathematics. I would certainly like to see further examination and studies on the role of problem-solving strategies in the teaching of mathematics.

Taloumis, Thalia. SCORES ON PIAGETIAN AREA TASKS AS PREDICTORS OF ACHIEVEMENT IN MATHEMATICS OVER A FOUR-YEAR PERIOD. Journal for Research in Mathematics Education 10: 120-134; March 1979.

Abstract and comments prepared for I.M.E. by ALAN OSBORNE,
The Ohio State University.

1. Purpose

The primary purpose of this study was to determine whether a predictive relationship existed between Piagetian area conservation tasks and mathematical achievement measures taken up to four years later. A second goal of the study was to contrast the contribution in prediction of the Piagetian task measures with other standardized test measures.

2. Rationale

Most previous studies of the predictive correlation between achievement in mathematics and the performance on conservation tasks have been limited to one-year studies typically conducted at the first-grade level. Few have contrasted the predictive power of performance on Piagetian tasks with other measures such as IQ or other standardized test measures. Better data concerning the predictive power of Piagetian tasks would allow researchers and curriculum developers better insights about the role of early exposure to Piagetian experiences in the school mathematics program.

3. Research and Design Procedures

The author used populations of first-, second-, and third-grade students from three Long Island, New York schools in a dissertation study during the early 1970s (Taloumis, 1975). Measures were taken on three area conservation tasks. This study reports the performance of these students as they matriculated through the next four years of schooling on Stanford Achievement Test subtests, the Otis-Lennon Mental Ability Test, and grades in mathematics. The initial dissertation study yielded two types of scores from Piagetian tasks, area conservation and area measurement. The relative value of the Piagetian task scores and the mental ability scores as predictors of achievement and of mathe-

matics class performance were assessed using two techniques: (1) correlation between predictor scores and individual criterion scores and (2) multiple regression analysis with hierarchical inclusion. The portion of the variance in criterion scores accounted for by the Piagetian measures was reported.

4. Findings

- a) The Piagetian area tasks had the most predictive power for Stanford subscale scores and for teacher marks in subsequent mathematics for the children tested in grade two as compared to those who were tested in grades one or three.
- b) The Otis-Lennon IQ Test had more predictive power than the Piagetian area tasks.
- c) The Piagetian area tasks had more predictive power for the concepts and applications subtests of the Stanford Achievement Tests than for the computation subtests.
- d) There was no decrease of overall predictability over the four years.

5. Interpretations

- a) The relative robustness of the Piagetian tasks as a predictor when used with grade two students, when compared to being used with grade one or grade three students, may be accounted for by the fact that grade two performance was distributed more normally. The tasks were too difficult for many first graders and too easy for a significant portion of the third graders.
- b) The relatively greater predictive power of the Piagetian tasks for the applications and the concepts subtests of the Stanford than for the computation subtest may be accounted for by noting that the applications and concepts subtests measure cognitive development more directly than the computation subtest.

Abstractor's Comments

The study provides interesting, important information relating performance on Piagetian tasks to children's subsequent achievement up to four years later. It is particularly noteworthy that the Piagetian

tasks all concerned a perceptual setting of a continuous rather than a discrete nature. The majority of Stanford test items concern content for which the selection of explanatory models and examples used by teachers and in texts are of a perceptually discrete nature. What kind of predictive power would the Piagetian tasks have had for items, such as multiplication of fractions, in which the likelihood of an instructional base of perceptually continuous objects is more likely? That is to say, would the role of area conservation tasks in contributing to subsequent achievement been revealed more completely if there have been an identification of content of the curriculum that was more likely to have been taught through instructional materials based on measurement concepts? This would be an effective followup to the present study.

Too little is gained in improving predictive power by use of the Piagetian tasks to supplement the IQ measures to warrant suggesting their incorporation in the testing program for children in the early elementary school. However, the strength of the instrument in contributing to understanding of the factors affecting learning of mathematics warrants its use in longitudinal studies, particularly if the outcome variables being studied have a high probability of being related to measurement or conservation of continuous quantities.

The author is to be complimented for the clear and concise reporting of a large quantity of statistics; the tables and graphs were well-designed for purposes of communication. The study was well-conceived and nicely reported.

References

- Taloumis, T. The relationship of area conservation to area measurement as affected by sequence of presentation of Piagetian area tasks to boys and girls in grades one through three. Journal for Research in Mathematics Education, 1975, 6, 232-242.

Trown, Anne. TEACHING STYLE, MATHEMATICS AND CHILDREN. Mathematics Teaching 82: 29-31; March 1978.

Abstract and comments prepared for I.M.E. by MERLYN J. BEHR,
Northern Illinois University.

1. Purpose

The purpose of two investigations reported was to determine whether there exists in mathematical learning any relationship between levels of "structure" in teaching and the abilities and personalities of pupils.

2. Rationale

The rationale for the work was cast in the philosophy of attribute-treatment interaction research. The question of whether exploratory or carefully guided instruction is best appears to be an unanswerable question unless individual characteristics of learners are considered. Teacher and researcher observation would suggest two opposing views: (a) exploratory teaching methods leave many children floundering from lack of guidance, and (b) structured teaching employing pupil packets and work cards have been so over-developed that there is little left for children to do.

It is indicated that, while results for research are generally equivocal, some useful clues are available from repeated classroom investigation. A consistent indication is that anxious, insecure, primary-age children do better in mathematical learning when they are provided with guidelines and the classroom is formally organized. Among older children consistent results seem to suggest that outgoing communicative extroverts are handicapped by the restrictions of closed expository teaching. Note is made of the observation that more research results indicate a greater potential for interaction between personality and instruction than between ability and instruction.

3. Research Design and Procedures

Two separate investigations, each involving 400 subjects, were conducted. The independent variables in each of the two studies were type

of instruction (exploratory or expository) and student ability (able vs. less able; extrovert vs. introvert; anxious vs. not anxious).

In the first study the type of instruction variable was exemplified by two types of instruction: principles explained and then applied to examples vs. use of examples to instigate a search for a principle. The subject matter, taken from the Midland Mathematical Experiments, involved the topic of vectors and was presented through carefully constructed worksheets. The subjects were twelve-year-old children.

The second study, involving ten-year-old children, exemplified a type of instruction variable similar to the first; however, the instructional material was presented by a teacher using objects, diagrams, and a flexible discussion format. The subject matter for this study concerned positive and negative integers.

For each of the studies the criterion variable was defined by a paper-and-pencil test.

4. Results

No main effects due to contrasting methods of instruction were found. Interaction effects of teaching method with ability were in each study found to be weak. Interaction effects between method of teaching and personality were, on the other hand, found to be strong. Among twelve-year-olds, extroverts were severely handicapped by worksheets which thrust rules upon them before they had a chance to investigate for themselves. Among the ten-year-old group, anxious children were found to score high when activity was used for verification of a principle but not when activity was used to encourage children to recognize patterns and principles for themselves.

5. Interpretations

Teachers' greatest service can be rendered to pupils by setting up open investigations which on some occasions and for some pupils are closed by the teacher. The teacher needs to be aware of the need for clear instructions, periods of consolidation, and summarizing conclusions. What is wrong with work cards is that they do not know when to stand back and allow the pupil to encounter mathematics.

Young, Courtney D., Jr. and Becker, Jerry P. THE INTERACTION OF COGNITIVE APTITUDES WITH SEQUENCES OF FIGURAL AND SYMBOLIC TREATMENTS OF MATHEMATICAL INEQUALITIES. Journal for Research in Mathematics Education 10: 24-36; January 1979.

Abstract and comments prepared for I.M.E. by DOUGLAS B. MCLEOD, San Diego State University.

1. Purpose

This aptitude-treatment-interaction (ATI) study was designed to replicate, improve, and extend earlier research by the authors. Phase 1 of the study tested the hypothesis that the use of figural or symbolic treatments would interact with aptitudes described as figural, symbolic, or general abilities. Phase 2 tested whether these same aptitudes would interact with different sequences of figural and symbolic material in the treatments. Phase 3 tested whether an interaction would occur between the use of figural and symbolic treatments and an aptitude called learning-to-learn.

2. Rationale

This study falls within the general framework of ATI research. The overall goal of this research is to identify ways to tailor instruction so that it matches the needs of individual students and optimizes their achievement.

3. Research Design and Procedures

Phase 1. Both the figural and symbolic treatment modes used programmed instruction on solving absolute value and quadratic equations and inequalities. Each mode took about one week of class time and used a guided discovery approach in presenting the material to students in grades 9, 10, and 11 who were enrolled in an elementary algebra course. The figural treatment (F) used geometric methods of solving the problems, and the symbolic treatment (S) used algebraic methods. Before instruction, students were assessed using six aptitude measures from the French kit and a pretest over the content of the treatments. At the end of the instruction in Phase 1, the students were assessed on achievement with

a "midtest." The "midtest" used the same mode as the subject's treatment.

Phase 2. There were three treatment sequences in Phase 2. In the first (F + S), the students who had used the figural mode in Phase 1 continued with the symbolic materials for a second week of instruction on the same content. The second treatment sequence (S + F) included students with a week of symbolic instruction in Phase 1 who continued with a second week of the same content using the figural materials. The third treatment sequence (ALT) was presented to students (not involved in Phase 1) who covered the same materials, but who alternated between figural and symbolic lessons for two weeks of instruction. The aptitudes used in Phase 2 were the same as those in Phase 1. A transfer test was administered at the end of each treatment in Phase 2, with a retention test one week later. The retention test, which included equal numbers of figural and symbolic items, was the same as the pretest.

Phase 3. This part of the experiment consisted of following the students from Phase 1 as they proceeded through the last half of Phase 2. The treatments consisted of either the figural (F) or symbolic (S) lessons, whichever the students did not have in Phase 1. The "midtest" administered at the end of Phase 1 is considered the aptitude measure for a "learning-to-learn" ability. The measures of achievement were the same transfer and retention tests that were used in Phase 2.

To summarize the three phases of the study, let X be the six aptitude tests and the pretest; let F and S be the figural and symbolic treatments, respectively; let M(F) and M(S) be the corresponding forms of the midtests; and let Y be the achievement measures (transfer and retention). Then the three phases of the study can be represented by:

Phase 1	X	F	M(F)
	X	S	M(S)
Phase 2	X	F+S	Y
	X	S+F	Y
	X	ALT	Y
Phase 3	M(F)	S	Y
	M(S)	F	Y

The 116 students who participated were randomly assigned to one of the three treatment sequences in Phase 2. This procedure also resulted in a random assignment to treatments in Phases 1 and 3.

4. Findings

The data were analyzed using regression techniques. If an aptitude was significantly correlated with the dependent variable in each treatment group, then that aptitude was tested to see if it produced a significant interaction. No other aptitudes (or combination of aptitudes) were tested to see if they produced any interactions.

When an interaction was found, it was classified as ordinal or disordinal. A disordinal interaction was said to occur when (1) the regression lines for two treatments crossed within the range of the aptitude scores, (2) the two slopes were different from zero, and (3) the difference between the two slopes was different from zero.

No significant interactions were found in Phases 1 and 2. In Phase 3, there was a significant disordinal interaction between the "midtest" scores and the treatment dimension. This interaction was still significant for the figural subtest of the retention test, but not for the symbolic subtest. No other interactions occurred.

5. Interpretations

Students who scored high on the "midtest" did better if, in their second time through the content, they used the figural materials. Students who scored low on the "midtest" did better on the second time through the content if they were in the symbolic group. Therefore, a student who studied the symbolic materials and learned little should be reassigned to study the same materials a second time. Students who were more successful on the "midtest" after using the symbolic materials could profit more by studying the figural materials for the second time through the content. Corresponding statements appear in the article for students who used the figural materials first.

An interaction found in the earlier study did not reappear here. The attempt at replication was not successful.

Abstractor's Comments

Substantial progress has been made in ATI research in recent years, and a large number of studies have produced interesting interactions. However, it is still a difficult area in which to work. The authors of this report should be commended for their willingness to work in a difficult area over an extended period of time. Some of their ATI research has been well done, but this particular study has serious weaknesses.

Phase 1

This phase tested the hypothesis that figural treatments are more appropriate for students who are strong in figural ability. A substantial number of studies have investigated this hypothesis with little success. Cronbach (1975) and Cronbach and Snow (1977) reject this hypothesis, and suggest instead that a more careful analysis of the processes involved in the aptitudes and treatments is required. Since the article makes no reference to any work of Cronbach and Snow that has appeared since 1969, it appears that the authors were not aware that more careful analysis of aptitudes and treatments was desirable.

A more serious weakness of the study is found in the statistical procedures. In the simplest type of ATI study, there is one aptitude, along with two treatments and a measure of achievement. The requirement of this study that an aptitude be a significant predictor of achievement in both treatments is not correct. It eliminates the kinds of interactions where the regression line has a steep slope in one treatment and a slope of zero in the other. This incorrect procedure has appeared in ATI research in mathematics education in the past, and has been described by Cronbach and Snow as inappropriate (1977, p. 287). The article's definition of disordinal interaction is also incorrect.

Phase 2

Phase 2 of the study suffers from the same weaknesses as Phase 1. In addition, there is no justification for the hypothesis that different sequences of figural and symbolic lessons will interact with any of the aptitudes. The rationale for Phase 2 is that frequently such concepts are taught using both figural and symbolic methods. This rationale is not relevant to the hypothesis being tested.

Phase 3

The third phase of the study produced a significant interaction between figural and symbolic treatments and an aptitude called "learning-to-learn." The learning-to-learn variable is an interesting one, and is considered important in ATI studies. Its importance is usually related to the need for "tuning," or familiarizing subjects with the kind of tasks that the treatments will include. In this study, however, it was treated like any other aptitude variable, except that it was apparently measured by two different tests (called the "midtest"), one figural and one symbolic.

If one is willing to assume that both versions of the midtest measured a learning-to-learn variable, then what are the implications of this significant interaction? If students score low on their form of the midtest, should we really have them continue in the symbolic mode, even if they have just finished working in that mode? According to the article, the answer is yes. I am not so sure. I am much more inclined to think that one significant interaction should be treated as a chance result, especially in a study with so many tests of significance and so little theoretical justification. And, as the authors point out, "confusion exists" in interpreting the interaction since the students had just been exposed to a very similar treatment.

The figure which presents the interaction in the article is incorrect. A correction will appear in a later issue of the journal.

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